Development of Pareto-Optimal Transit Routes in Afghanistan through Mathematical Algorithms

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Abstract. Afghanistan is geo-strategically in an important zone in South and Central Asia with six neighbouring countries, but currently lacks of modern infrastructure. We present the construction of optimal transit routes in Afghanistan through mathematical optimization. Basically there are three different optimization goals a) shortest route w.r.t. the distance, b) the cheapest route w.r.t. the construction cost, and c) the most convenient route w.r.t. the elevation change. It is possible to combine two objectives by considering the Pareto front. For the design and modelling of the routes, a computer program named **»CONTRA«** (**C**omputing an **O**ptimal **N**etwork of **T**ransit **R**outes through mathematical **A**lgorithms) was developed. As a demonstrator example, we compute Pareto-optimal routes between two Afghan cities.

1 Aim and Idea

Afghanistan is located geographically in the center of Asia and is surrounded by six neighboring countries. The country has a great potential to transform into a "logistical crossroad" in South and Central Asia. A huge problem which prevents the transportation in this region is the weak infrastructure in Afghanistan. The aim of this study is to develop optimal transit routes in Afghanistan by mathematical optimization methods. In this paper, the focus is on the shortest path problem between two cities. The shortest path problem belongs to the class of graph problem and deals with the issue of how to compute an optimal route between two nodes or points (start and end point). There are three route optimization targets. A route can be considered optimal regarding the

1. route length (shortest),
2. construction cost of the route (cheapest),
3. elevation change (most convenient).

For the design and modelling the routes, a computer program named »**CONTRA**« (**C**omputing an **O**ptimal **N**etwork of **T**ransit **R**outes through mathematical **A**lgorithms) was developed. In order to calculate the costs of a route segment, e.g. 1 km, several factors and data are taken into account, in particular the national land use of Afghanistan and the elevation profile of the terrain (topography). CONTRA transforms the input data into a weighted grid graph.

2 Input Data

To determine a route automatically with the help of mathematical optimization, the following input data has to be provided to the program CONTRA:

1. Coordinates of start and destination points (given as latitude and longitude).

2. National land cover of Afghanistan in shape files (currently from date 1997) of the organization AIMS, originally of the Afghan Geodesy and Cartography Head Office [1]. The whole country is separated in polygonal

shapes that describe the respective type of land, see Figure 2.

3. Topographic representation of Afghanistan, the SRTM data of USGS / NASA [2]. The resolution of the SRTM data for Afghanistan is 14000 x 18001 pixels. As an example, in Figure 1 the area of the province of Kabul in Afghanistan is depicted.

4. The costs of construction and maintenance of a road. According to reports of the Asian Development Bank the construction of a (two-lane) road in Afghanistan on average is approximately 1 Mio. USD per km [3]. The cost amount can be dependent on the height and the land surface on which it is built [4]. For this research the assumption is an estimated cost ratio among the different land covers, and thus determine the construction costs of the routes. In general, it is possible to modify the estimated cost ratio.

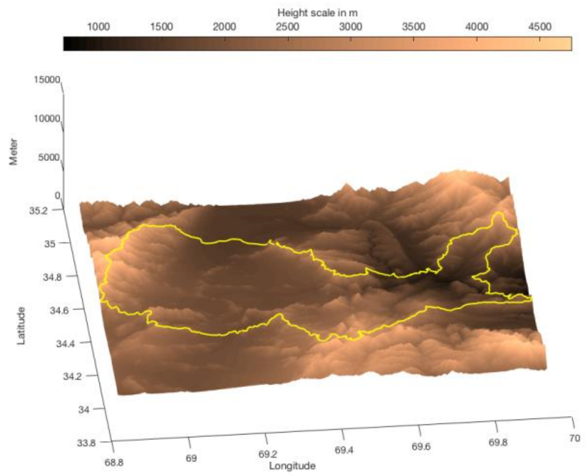

Figure 1. The topography of the province of Kabul in Afghanistan. The color spectrum from dark to light indicates the height of the terrain.

3 Shortest Path Problem on Terrain and Multi-Objective Optimization

The problem of the construction of optimal routes on a terrain in Afghanistan leads to the shortest path problem. A shortest path is a route that is minimal with respect to the sum of all costs of all segments that are used in the entire route. It is assumed that the cost per segment can be determined from the construction cost, the length and the height variation. To solve the shortest path problem Dijkstra's algorithm [5] is used.

Besides focusing on a single optimization goal, there are several conflicting objectives to consider. This could be, for instance, minimizing the total length of the route as well as the construction cost. These two can be in conflict, because a shorter route may go through more difficult terrain that a slightly longer route would have avoided, and thus turns out to be less costly. The three objectives route “length”, “construction cost” and “elevation variation” are conflicting with each other. Multi-objective optimization (Pareto optimization) is an area of multi-criteria decision making, that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously (as minimizing the length of the route as well as construction costs) [6]. In particular, there is no route that is simultaneously optimal for all three. Thus one has to analyze the trade-off between them. A Pareto optimum is the result of a multi-objective optimization. In CONTRA, a user is able to enter the coordinates of start and end points and then run this program. It computes the three routes (the shortest w.r.t. the distance, the cheapest w.r.t. the construction cost, and the most convenient route w.r.t. the elevation change), and visualizes them on the map (see Figure 2). Furthermore, CONTRA is able to combine two goals, e.g., minimizing the length of the route as well as construction costs and to find the Pareto optimum.

3 Calculation of an Optimal Route

As an example, we compute routes between the city of Kabul and the city of Jalalabad. First, using CONTRA, three different optimal routes are calculated with the above single objectives a) the shortest (red), b) the cheapest (blue), and c) the most convenient routes (white-purple), see Figure 2.

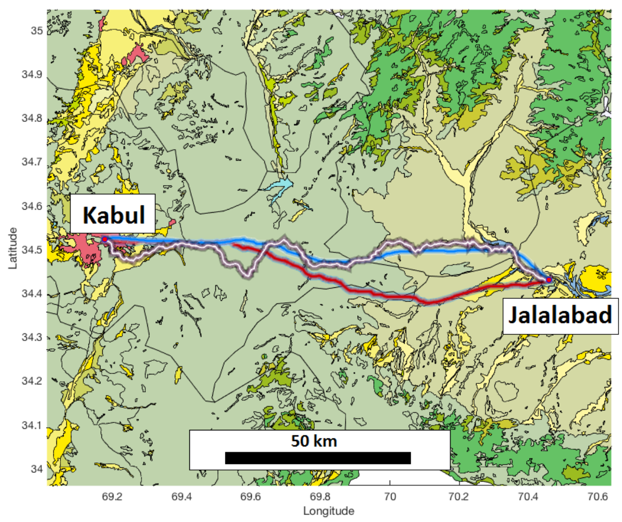

Figure 2. Three single-objective optimal routes: shortest (red), cheapest (blue), and most convenient (white-purple).

Table 1 represents the results of the mentioned routes. The columns of this table describe a) the construction costs of the routes in million USD, b) the lengths of the routes in km and c) the absolute elevation changes of the routes, i.e., the sum of all height changes from the starting point to the ending point along the route.

Table 1. The result of the three routes regarding the cost, length and absolute elevation change.

Route	Cost in mio. USD	Length in km	Absolute elevation change in m
Shortest (red)	140.30	**128.11**	7,350
Cheapest (blue)	**130.58**	132.46	8,594
Most convenient (white-purp.)	189.41	165.88	**5,051**

Each of these three routes has a certain length and elevation profile. Figure 3 shows the height profile along these three routes.

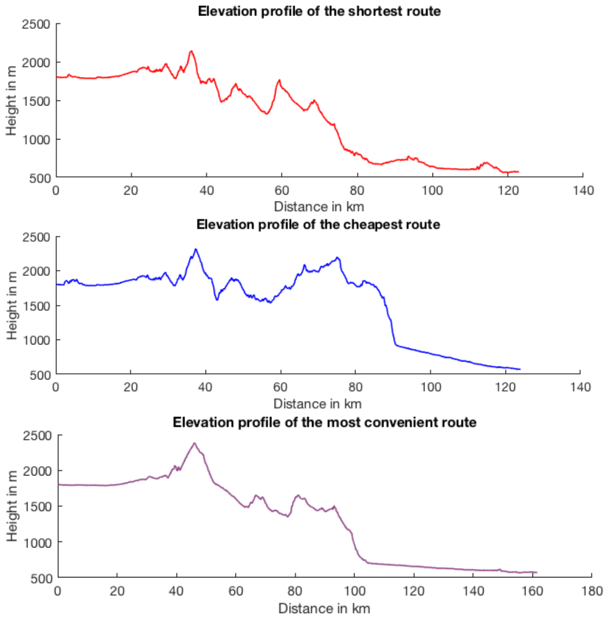

Figure 3. The height and length diagram of the shortest, the cheapest, and the most convenient route.

Figure 4 shows the Pareto front of the routes regarding the “distance” and “construction costs” from the City of Kabul to the City of Jalalabad. This chart shows on the horizontal axis the length of the routes in km and on the vertical axis the construction cost of the routes in million USD. The small red circles represent specific routes between these two cities. The leftmost circle represents the shortest route and the rightmost circle represents the cheapest route. The circles that lie between these two extrema circles are other optimal routes that are a combination of the shortest and cheapest route from Kabul to Jalalabad.

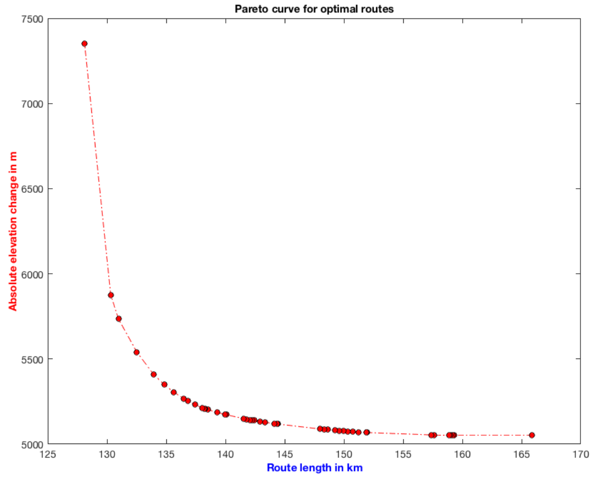

Figure 4. The Pareto front of the shortest and cheapest routes regarding length and construction cost from Kabul to Jalalabad.

As the project acronym CONTRA indicates, our future work is to extend these point-to-point connections to automatically design large ***networks*** that connect several cities with optimally located transit routes.

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